Modelling of the Rubber-metal Suspension Components in the Railway Vehicle Dynamics Simulations

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The paper features several one-dimensional models to represent the rubber springs used in the dynamics simulations for the railway vehicles. Similarly, it includes a description of a model for the illustration of the mechanical behaviour of a mix coil spring - rubber employed in the secondary suspension of the passenger cars. The principle of this model relies on the overlapping of three components – elastic, viscous and the dry friction. The dependence of the force developed in the mix coil spring - rubber on the frequency and amplitude of a harmonic excitation is highlighted via the numerical simulations.

Keywords: suspension, railway vehicle, rubber models, coil spring – rubber, friction

The suspension of the railway vehicles is an essential element in providing the dynamic performance required by a modern transportation means - safety, ride quality and ride comfort. The increase in the rail velocity and a surging exigency for the comfort of the passengers have entailed the adoption of suspension components that will trigger an as good as possible damping of the oscillations and of the noises conveyed to the vehicle. Thus, the rubber coils have been introduced in the suspension of the railway vehicles as a result of the advantages in their use. Firstly, it is about a stiffness and damping in a single component, which rise along with the excitation frequency and the decrease in the amplitude [1, 2]. Likewise, in comparison with the metallic springs, the rubber ones have diverse benefits, namely a smaller suspension weight and its constructive simplification, a better comfort for the passengers by keeping the oscillation frequencies at values under 3 Hz and provision of a good sound proofing, irresponsiveness to the short-time overloading and the low maintenance costs [3].

The rubber springs used in the suspension of the railway vehicles consist of the rubber blocks interleaved with or reinforced by steel plates. The structure of the suspension in the passenger cars can also include metal – rubber mix suspension elements, where the metallic coils will work together with the rubber springs [4].

In the context of the simulation in the dynamics of the railway vehicles, the issue of the suspension components for this transportation has been the object of ample research studies in the last years [2, 5, 6]. Generally speaking, the complexity degree of the model of the suspension elements is established as a function of the precision asked for the model of the vehicle or of the train. The more complex the model, the closer the results will be to reality, but more difficult the drawing of general conclusions regarding the basic phenomena of the dynamic behaviour of the vehicle. A component of the suspension can be modelled in detail by considering the geometric shape and the properties of the material of its construction, also including a potential pre-loading. In many cases, such models are complicated and, therefore, difficult to be implemented in the simulation programs for the vehicle dynamics and they require long simulation times. Quality and quantity information can be obtained from less complex models. It should be taken into account the fact that the too simple suspension models and/or inaccurate parameters of the model can lead to erroneous results of the simulations for the dynamic behaviour of the railway vehicles.

The object of this paper is focused on the modelling of the suspension rubber components used in the dynamics simulations of the railway vehicles. The context features typical models of the rubber springs, as well as an original model for a mix coil spring – rubber suspension element. The model of the coil spring – rubber unit underlies on a non-linear dynamic model for rubber springs [7-9], which best reflects the mechanical behaviour of the rubber suspension elements in the dynamics of the railway vehicles and it also provides a good concordance between the theoretical and experimental results. The main properties of the model, namely the dependence of its response on the excitation amplitude and frequency, are made obvious via certain applications of numerical simulation.

One-dimensional models for the rubber springs

In many instances, the modelling of the rubber springs uses one of the simplest rheological models, i.e. Kelvin – Voigt model, made up of an elastic element working in parallel with an element of viscous damping, where the features of the two elements are linear [10]. The Kelvin-Voigt model is regarded as acceptable in a very limited frequency range. The increase of stiffness along with frequency is significant and such models are often not representative for rubber.

A typical model for the rubber springs is shown in figure 1, where an elastic element is in series with a viscous friction damper (Maxwell model). For this model, stiffness takes values within the interval (k, $k + k_s$) and it has a monotonous increase along with the excitation frequency [2]. In order to have a better frequency dependence of the model, further Maxwell elements can be added in parallel [11].

For the above models, the hysteresis effect tends to zero when the excitation frequency does the same. Such behaviour is not generally made obvious by measurements. Even for low frequencies, there is a hysteresis effect coming from either the rubber itself

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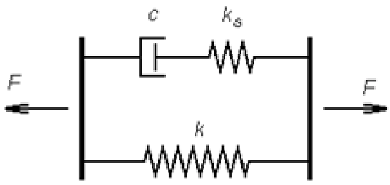

Fig. 1. One-dimensional model for rubber.

(internal friction) or the friction between rubber and the metallic elements to which it is volcanized to (external friction). The internal and external friction will rise the stiffness of the rubber springs, mainly for movements of low amplitude. The increase tendency of the rubber stiffness at high frequencies is also due to the low amplitude in the movement at high frequencies [2].

To expand the above-described models and to take into account the mentioned phenomena, the friction should be represented in the model. Such model shows in figure 2. This is the model suggested by Berg [7], which relies in the overlapping of three component forces working in parallel, where each of them contributes to the total deformation force of the rubber coil. It is about the component of the elastic force (F_e), the viscous force (F_v) and the dry friction component (F_f). The elastic force component is linear and models the rubber elasticity property. When introducing the viscous force, the increase in stiffness triggers a similar response in the frequency, as well as the rate-dependent hysteresis. The inclusion of a friction force means an increased stiffness at small displacement amplitudes as well as rate-independent hysteresis.

The friction part of the model in figure 2 can be a Coulomb friction element in series with an elastic element. As for the viscous part, more friction components in parallel can be added so as to model the adhesion/sliding succession, as shown in figure 3 [12].

The Berg model very well expresses the mechanical behaviour of rubber suspension components in rail vehicle dynamics. It should be noticed that the Berg model provides a good concordance between the theoretical and experimental results and represents a reasonable compromise between accuracy and computational effort [7 - 9].

In terms of the bi-dimensional or tri-dimensional models of the rubber springs, they are usually derived from the overlapping of the one-dimensional models [2].

The modelling of the mix coil spring-rubber suspension elements

The figure 4 shows a mix coil spring - rubber used in the secondary suspension of the Y 32 bogie which certain passenger rail vehicles are fitted with. This combination of steel coil spring and rubber benefits from a high stiffness on the vertical direction and a low stiffness on the transversal one. The static characteristic of the mix on the vertical direction is featured in figure 5. This characteristic is noticed to be of a non-linear type with a stiffness step variation. The stiffness of the mix increases along with the applied loading, due to the rubber deformation.

From the perspective of the simulation in the dynamic behaviour of the railway vehicle, what holds the attention

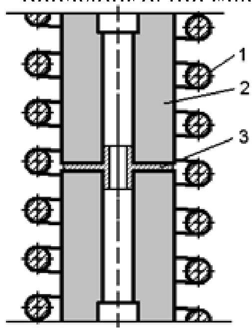

Fig. 4. Mix coil spring - rubber: 1. coil spring; 2. rubber; 3. intermediary element

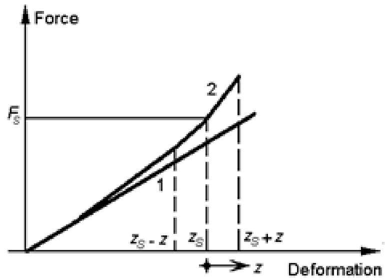

Fig. 5. The static characteristic of the mix: 1. the characteristic of the steel spring; 2. the characteristic of the coil spring + rubber

is how the mix behaves when it has a deformation z around the point z_s, corresponding to the static load F_s. In dynamic behaviour, the properties of the rubber and of the non-linear elastic element trigger an asymmetric hysteresis cycle, which is also present during a very low frequency.

The recommended model for the representation of the coil – rubber unit is featured in figure 6. This model relies on Berg's (fig. 2) as it comprises the same three components, working in parallel, similar with the initial model. The elastic force component is thus described by an elastic element that has a variable stiffness k_e, unlike the Berg model. It both includes the elastic force in the coil spring and the one in the rubber element. As for the viscous force component, this is represented by a Maxwell system containing an element of viscous damping of constant c_v in series with an elastic element of stiffness k_v.

The component of the friction force, whose modelling is based on Coulomb's dry friction theory. As a principle, to avoid the issues emerging during the numerical simulations due to the fact that Coulomb model is non-smooth, multi-valued and non-differentiable (the force-displacement curve, fig. 7, a), a linear spring in series with a friction slider is used (fig. 8) [2]. The result will be the force-displacement curve, shown in fig. 7, b. The present friction model can be seen as a 'smooth Coulomb friction force' [7 - 9].

In the presented model (fig. 6), the spring in series with the friction elements has a non-linear characteristic. Hence, the non-linearity of the model comes, on the one hand, from the component of the elastic force developed in the coil spring and rubber element unit and, on the other hand, from the component of the friction force emerging in the rubber element.

For a displacement z in relation to the equilibrium position of the system under the action of the static force,

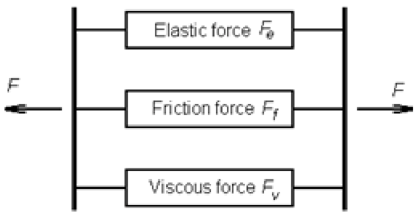

Fig.2. The principle of the one-dimensional model for the representation of the rubber springs [7]

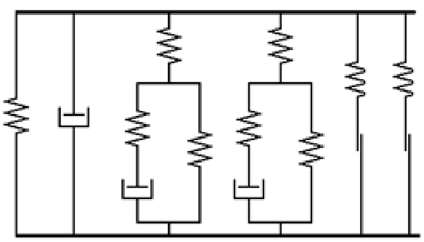

Fig. 3. One-dimensional model for elastomeric components [12]

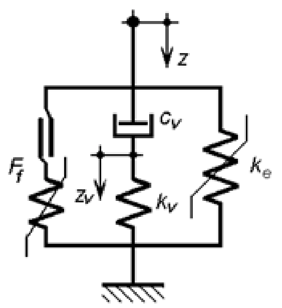

Fig. 6. Model for mix coil spring – rubber

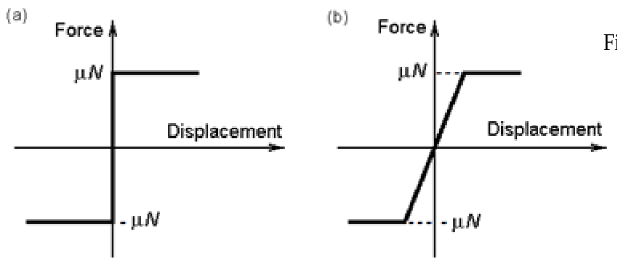

Fig. 7. Force–displacement curve: (a) Coulomb friction model; (b) Coulomb model with spring in series

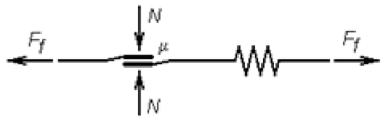

Fig. 8. Friction element with spring in series

the total force F is equal with the sum of those three forces emerging in the components of the model:

$$F = F_e + F_v + F_f \, . \qquad (1)$$

The elastic force is calculated as a function of the direction of the displacement z, while considering the variation in the stiffness of the elastic mix (with $k_{e1} > k_{e2}$)

$$F_{e1} = k_{e1} z \, , \text{ for } z > 0; \qquad (2)$$

$$F_{e2} = k_{e2} z \, , \text{ for } z \leq 0, \qquad (3)$$

The viscous force is determined as such

$$F_v = c_v(\dot{z} - \dot{z}_v) = k_v z_v \, . \qquad (4)$$

The friction model depends on the displacement z of the system, as well as on the reference state, defined by the coordinates of the turning point in the cycle running, (z_r, F_{fr}) [7]. Besides the reference state, two more parameters of the friction force component model are defined, namely the maximum friction force F_{fmax} and the displacement z_2 for which the friction force is half the value of the maximum friction force $F_f = F_{fmax}/2$, when starting from the initial reference state (0, 0) and reaching $z = z_2$.

The friction force is defined in relation to the reference state, as such:

$$F_f = F_{fr}, \text{ for } z = z_r; \qquad (5)$$

$$F_f = F_{fr} + \frac{z - z_r}{z_2(1-\alpha) + (z - z_r)}(F_{f\,\max} - F_{fr}), \text{ for } z > z_r \, ; \qquad (6)$$

$$F_f = F_{fr} + \frac{z - z_r}{z_2(1+\alpha) - (z - z_r)}(F_{f\,\max} + F_{fr}), \text{ for } z < z_r \, , \qquad (7)$$

where the parameter $\alpha = F_{fr} / F_{fmax}$ and its values ranges from -1 and 1.

The basic properties of the model herein, namely the dependence of its response on the amplitude and frequency of an imposed displacement harmonic excitation, can be highlighted by numerical simulations, based on the above relations. For this purpose, the following model parameters are considered: k_{e1} = 700 kN/m, k_{e2} = 500 kN/m, z_2 = 0.3 mm, F_{fmax} = 0.4 kN, k_v = 150 kN/m and c_v = 3 kNs/m. As for the amplitude of the displacement, typical values will be taken into account for the secondary suspension of a railway vehicle.

The figure 9 shows the force – displacement characteristic of the model at a harmonic excitation with 8 mm amplitude. The excitation values of 1, 7 and 12 Hz are to be found within the interval of 0 ... 20 Hz, specific to the vibrations in the railway vehicles. The diagram (a) shows the elastic force that has the property to be independent from the excitation frequency. The characteristic of the elastic force is in the shape of a broken line due to the variation in stiffness, which is larger for positive displacements and lower for negative ones.

The friction force – displacement diagram (diagram (b)) is symmetrical and independent from the excitation frequency. The variation of the friction force has the form of a smooth curve, except for the points of maximum amplitude that are angular points.

The viscous force has also a symmetrical characteristic but its amplitude rises along with the excitation frequency (diagram (c)) unlike the friction force. Indeed, the amplitude of the viscous force is F_{vo} = 0.15 kN for 1 Hz; it is also 0. 79 kN and 1.00 kN, respectively, at 7 Hz and 12 Hz.

The properties of the elastic, friction and viscous forces influence the shape of the total force (diagram (d)). The diagram is a smooth curve with angular points in the middle

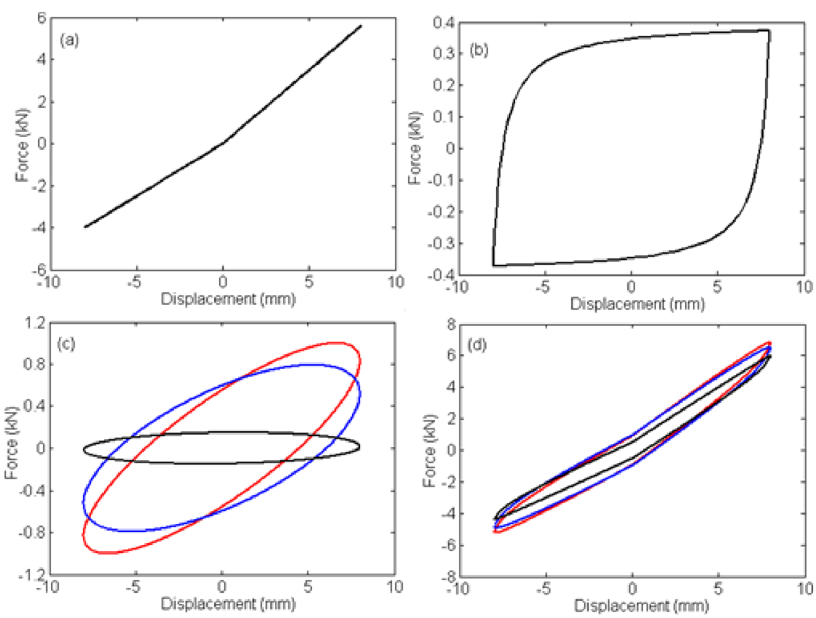

Fig. 9. The influence of the excitation frequency upon the force – displacement characteristic:
(a) elastic force (F_e); (b) friction force (F_f); (c) viscous force (F_v); (d) total force (F): -, 1 Hz,-, 7 Hz, -, 12 Hz.

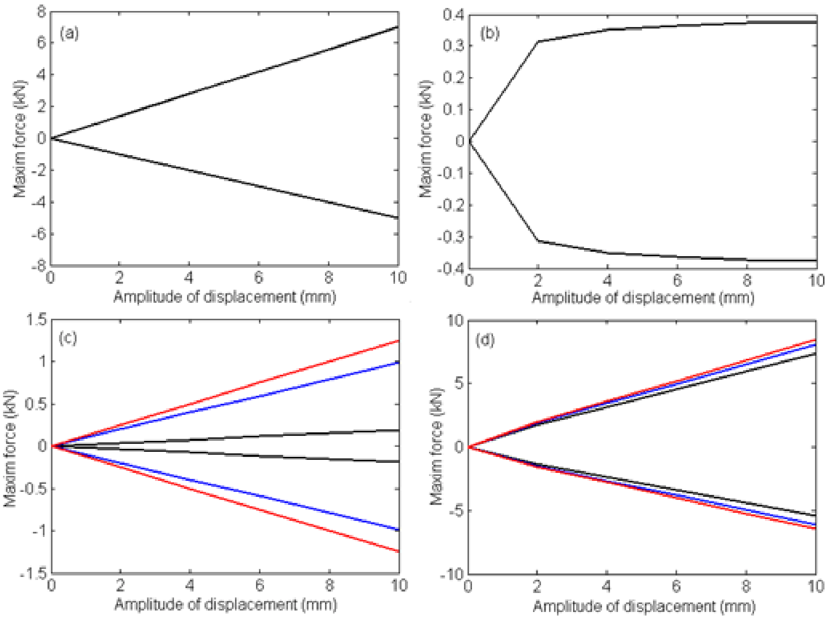

Fig. 10. The influence of the excitation amplitude upon the maximum force: (a) elastic force (F_e); (b) friction force (F_f); (c) viscous force (F_v); (d) total force (F): -, 1 Hz, -, 7 Hz, -, 12 Hz.

of the cycle and at its ends, and the amplitude of the total force goes up along with the excitation frequency.

The influence of the excitation amplitude upon the maximum force of the mix coil spring – rubber is shown in figure 10. There will be considered values of the amplitude between 0 and 10 mm for the three values of the excitation frequency (1 Hz, 7 Hz and 12 Hz). The diagrams feature the maximum values of the elastic force (diagram (a)), friction force (diagram (b)), viscous force (diagram (c)), as well as of the total force (diagram (d)).

The elastic force and the viscous force are noticed to have a linear variation with the excitation amplitude, while the friction force has a sudden increase for displacements smaller than 2 mm, after which its variation is very low. Since the percentage of the friction force is small, the variation of the total force as a function of the excitation amplitude is practically linear.

Conclusions

The simulation of dynamics in the railway vehicles represents a basic tool in research as being used since the designing stage in order to estimate the dynamic behaviour of the railway vehicle and the optimization of its dynamic performance and in the investigation of the problems arising during exploitation. The potential and success of vehicle dynamic simulations greatly depend on how the mechanical properties of the vehicle in general and its components, in particular, are modelled. The suspension components hold an important place in the dynamic behaviour of the railway vehicles and, hence, the adopted models are critical for the precision of the vehicle models.

The paper herein introduces several typical models used for the rubber suspension elements in the dynamics simulations for the railway vehicles. Similarly, there is presented an original model for the depiction of the coil spring – rubber suspension elements that are to be found in the secondary suspension of certain passenger cars. The basic principle of the suggested model consists in the overlapping of non-linear elastic component modelling the characteristic of progressive elasticity with the unit load, with a viscous component and a friction component with a non-linear characteristic that models the internal and external frictions of the rubber suspension elements.

The results of the numerical simulations derived from the model implementation in the computer package Matlab highlight a series of its properties. It is about the dependence of the model response to the amplitude and frequency of a harmonic excitation of an imposed displacement type. Thus, the conclusion is that the amplitude of the force developed in the coil spring – rubber rises along with the excitation force. Moreover, the maximum force practically increases linear for excitation amplitudes that are higher than 2 mm.

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